

**MAGNETIC FIELDS FROM DISPLACEMENT**  
**CURRENT DENSITIES GENERATED BY**  
**THE CROSSED FIELD ANTENNA**

presented by

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## MAGNETIC FIELD FROM DISPLACEMENT CURRENT

Maxwell's modification to Ampere's Law is as follows:  $\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$ . This states that the curl of the magnetic field is equal to the current density, in amperes per square meter, plus the time derivative of the electric field displacement charge density, also in amperes per square meter.

For evaluation purposes this is equivalent to  $\nabla \times \mathbf{H} = \mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$ , where  $\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$ .

In the absence of conduction current density the  $\mathbf{J}$  term disappears leaving only the time derivative of the electric field multiplied by  $\varepsilon_0$  to create the magnetic field. If we assume a constant sinusoidal electric field we have  $\varepsilon_0 \omega E_{\max} \cos \omega t$  for the magnitude of the time derivative of the displacement charge density. For engineering purposes the magnitude of the displacement current density is  $\varepsilon_0 \omega E_{\text{RMS}}$ , amperes per square meter, where  $\omega$  is  $2\pi f$ , the radian frequency, and  $E_{\text{RMS}}$  is the RMS field in volts/meter.

If the displacement current density is multiplied by the area of the plate  $\pi r^2$ , where  $r$  is the plate radius in meters, to get the current magnitude in amperes, and the voltage from the plate to the ground plane,  $V$ , divided by the distance  $z$  is equal to the constant field,  $E_{\text{RMS}}$ , we have  $\mathbf{I} = \pi r^2 \varepsilon_0 \omega V / z$ .

The expression relating the displacement current from the plate to ground to the voltage from the plate to ground,  $\mathbf{I} = \pi r^2 \varepsilon_0 \omega V / z$ , is simply a derivation of ohms law from the point of view of displacement.  $(\pi r^2 \varepsilon_0) / z$  is the capacitance of the plate  $C$ , in farads, and  $\omega C = 1 / X_c$ , where  $X_c$  is the reactance of the plate. Of course the equation of continuity and Kirchoffs law requires this equivalence.

The plate used in a typical MF crossed field antenna is 0.6 meters above the ground plane and 2 meters in radius. These dimensions are very small compared to a wavelength at medium frequencies. Thus a quasi-static analysis can be employed, using the Biot-Savart law, to compute the magnetic field from the displacement current density between the plate and an assumed perfectly conductive ground plane. Because the spacing of the plate above the ground plane and the diameter of the plate are so small compared to a wavelength at medium frequencies we can ignore fringing effects and assume constant axial displacement current density magnitude between the plate and ground, and constant axial displacement current density magnitude across the diameter of the plate.

The Biot-Savart law is the magnetostatic analogue of Coulomb's law for electric fields. For a current flow between two points the magnitude of the magnetic field  $H$ , amperes per meter, is given by Biot-Savart as  $\frac{\mathbf{I}}{4\pi R} (\sin \theta_1 - \sin \theta_2)$ , where the  $\theta$  is the angle from the observation point at a distance  $R$ , in meters, from the center of the current flow to each of the respective points between which the current flows and  $\mathbf{I}$  is the current in amperes.

If the distance between the plate and ground is  $z$ , the sine of the angle from a point situated at half the height of the plate at a distance  $R$  is close to  $z/2 \times 1/R$  for an  $R$  that is large compared to  $z$  (say ten times). Since one of the angles,  $\theta_2$ , is negative, the sum of the two sine functions is  $z/R$ . The magnetic field is then given as  $\mathbf{I}(z) / 4\pi R^2$ .

When we insert the expression for the current,  $\mathbf{I} = \pi r^2 \epsilon_0 \omega V / z$ , in the expression for magnetic field,  $H = \mathbf{I}(z) / 4\pi R^2$ , we have  $H = \frac{r^2}{4R^2} \epsilon_0 \omega V$  amperes per meter, which is equivalent to

$$H = \frac{\epsilon_0 \pi f r^2 \omega V}{2R^2}.$$

The magnetic field from the displacement current density is therefore determined by the voltage across the plate, V, the frequency, f, in hertz, and the distance to the observation point, R. For r = 2 m, R = 10 m, and f = 1.16 MHz.,  $H = (6.45 \times 10^{-7}) V$ .

The major claim made by the inventors of the crossfield antenna is that the electric field from the cylindrical portion of the antenna, or pipe, couples with the magnetic field created by the displacement current density flowing from the plate to ground such that  $\mathbf{E}/\mathbf{H} = 120 \pi$  and a far field plane wave is thereby created. We can apply the relationship between the plate voltage and the magnetic field from the plate displacement current density to determine the plate drive voltage required to obtain the plane wave ratio,  $120 \pi$ , between the electric field from the cylinder and the plate generated magnetic field. The electric field computed for the crossfield antenna at 10 meters is 300 volts/meter @ 1 kilowatt. If plane wave conditions,  $\mathbf{E}/\mathbf{H} = 120 \pi$ , are assumed to exist at 10 meters, the required voltage from the plate to ground is  $(300) / [120 \pi \times 6.45 \times 10^{-7}]$  or  $1.23 \times 10^6$  volts!

In other words 1.23 megavolts must be applied across the 0.6 meter from the plate to ground for the antenna to work as claimed and produce  $\mathbf{E}/\mathbf{H} = 120 \pi$  for plane wave synthesis for an input power of 1000 watts!

It is informative to determine the plate drive voltage that is required to generate the same magnitude of magnetic field from the plate displacement current density that is generated by the conduction current flowing in the cylinder. The near computed magnetic field 10 meters from the conduction current portion of the crossfield antenna, i.e. the pipe, is 0.304 amperes / meter @ 1 kilowatt. To create the same magnitude of magnetic field by the plate displacement current density that is generated by the conduction current in the cylinder, there would have to be about 470,000 volts across the plate!

## CONCLUSION

The final conclusion is that, for operation with 1 KW into the pipe section of the CFA, it would take **1.23 megavolts into the plate with a current of 1667 amperes** to provide enough magnetic field from the plate **to provide the plane wave synthesis** claimed by the inventors of the CFA. The pipe section of the CFA also creates a magnetic field and is much more efficient at producing magnetic field than the plate. It would take 471,000 volts with a current of 636.7 amperes into the plate for the plate to produce a magnetic field of the same intensity as that resulting from 1 KW into the pipe.

The conduction current into the capacitor must equal the displacement current. Equal currents produce equal magnetic fields and a 0.6 meter wire that replaced the plate must, by the requirements of continuity, produce the same magnetic field as the plate. The pipe is much greater in length than the distance from the plate to ground and will produce correspondingly greater magnetic fields for equivalent currents flowing in both sections of the CFA. The contribution of the magnetic field from the plate to the total field, being so small, can safely be ignored compared to the magnetic field from the pipe. The question also arises of how the plate can be fed without the magnetic field in the feed wire canceling the magnetic field from the plate since the current in the feed wire runs in the opposite direction to the current in the plate.