RECIPROCITY AND MOMENT METHODS APPLIED TO PREDICTING RADIATED EMISSIONS

James B. Hatfield

Hatfield & Dawson Consulting Engineers

Seattle, Washington, USA

Conductors within electrical and electronic devices act as antennas for the transmission and reception of RF energy. EMC measurements are frequently required to determine the impact of RF fields upon such equipment (RF immunity tests). It is usually required that measurements also be made of the RF fields created by currents flowing in the devices (Radiated emissions tests). There are conditions under which the principle of reciprocity can be used to relate the two measurement procedures. A complication arises from the fact that RF emissions at lower UHF frequencies can be near field phenomena at distances of ten metres, or more, from the device under test.

Introduction

To begin with we shall confine ourselves to the terminals of the transmitting and receiving antennas. The devices and antennas discussed in this paper are shown in greatly simplified schematic form to illustrate the principles involved. (i.e. I do not deal with the complexities of interconnecting cables in actual equipments upon which tests are performed, and the conical dipole usually used for measuring RF fields is represented by a thin wire equivalent antenna. This makes the job of modeling the geometry for MININEC computations much simpler.) When RF Immunity tests are performed currents are usually measured at a wire junction or on the wire at the input or output of a sub assembly in the Device Under Test (DUT). This interconnecting wire in the DUT is called Antenna #1. A standard dipole is usually used to measure the RF fields from the DUT and to determine the field intensity of the RF fields used to perform RF immunity tests upon the DUT. This is antenna #2. Antenna #2 can also be used to produce the RF fields for immunity tests (see below), though this is usually the function of Antenna #3. These antennas are shown in Figure One. It is assumed that all antennas are over a uniformly conducting ground plane.

The antenna sizes, types, and the distances between them (Figure One) come from an attempt I made in 1967 to apply reciprocity to a specific set of tests being conducted at that time.

Later in this paper calculations for a measurement distance of three metres will be shown.

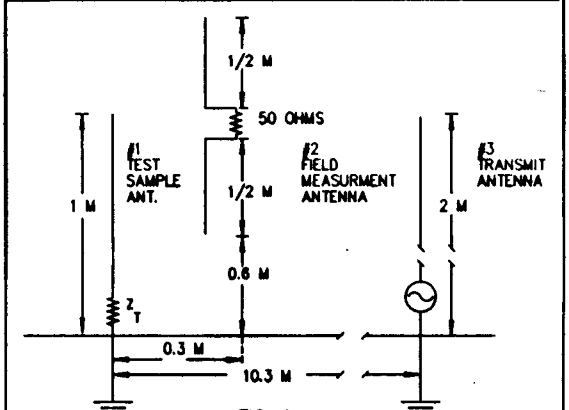

FIG. 1
ANTENNAS AND TEST SETUP

Calculating Radiated Emissions

The principle of reciprocity [1] states that

$$\frac{E_2}{I_1} = \frac{E_1}{I_2} \qquad (1)$$

This means that if you drive the terminals of antenna two with a voltage E_2 and you get a current I_1 at the terminals of antenna one, and then reverse the process by driving antenna one with E_1 volts and producing a current I_2 at antenna two, the ratios of the voltages to the currents in each case will be the same. If the voltages are identical the currents will be identical.

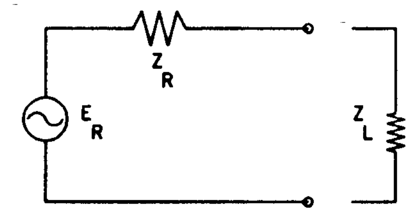

FIG. 2
EQUIVALENT CIRCUIT FOR RECEIVE ANTENNA

For this relationship to hold the generator and load impedances must be the same in each case. If the generator impedances are different then the actual terminal voltages of the driven antennas must be known. If the load impedances are different a correction factor can be derived from consideration of Figure Two [2]. The relationship between the receive antenna terminal voltage and the load current [3] is given by

$$I_1 = \frac{E_r}{(Z_r + Z_1)} \qquad (2)$$

From these expressions the current at the terminals of the measurement antenna (Antenna #2 terminated in 50 Ohms) produced by the RF emissions of the DUT (Antenna #1) is

$$I_2 = \frac{E_1\,I_1}{E_2}\;\frac{Z_r + Z_1}{Z_r + 50} \qquad (3)$$

where:

I_2 = Current at the terminals of Antenna #2 produced by the fields from the DUT.

E_1 = The voltage at the point in the device under test where the immunity currents are measured. This is the location of the terminals of Antenna #1.

I_1 = The current in the DUT produced by the fields used for immunity testing. At the terminals of antenna #1.

The voltage at the terminals of antenna #2 to produce the fields for immunity tests.

Z_r = The impedance of Antenna #2 at the test frequency.

The load impedance at antenna #1 terminals of the DUT at the test frequency when immunity tests are performed.

There are several conditions on the use of this equation:

(1) the impedance presented by the DUT, as an antenna load, to its wire or cable during RF immunity testing must be known as a function of frequency. This is the load at the antenna #1 terminals where the immunity currents are measured.

(2) The impedance of the measurement dipole, Antenna #2, must be known as a function of frequency.

(3) The same antenna, Antenna #2, must be used to generate the fields used for immunity testing as is used to measure the fields generated by the DUT. Antenna #2 must be the same distance from the DUT in both cases and in the same relative orientation.

This equation in terms of dB is:

FIELD FROM DUT (dBu) = (Antenna #2 factor) + E_1(dBu) + I_1(dBua) + 10 log(Z_r + Z_1) - E_2(dBu) -10log(Z_r + 50). (4)

Where broadband RF emissions are concerned the device impedance (Z_1) and terminal voltage (E_1) must be approximated or measured over some interval of spectrum.

Fields and Currents Computed by MININEC

Thin wire models of the antennas shown in Figure One were devised so that antenna currents and fields could be computed using the program MININEC [4]. Wire radii were chosen to be small (0.0032 metres) in relation to the segment lengths (0.11 metres) and the wavelength of the highest frequency considered (0.67 metres at 450mHz).

Computations were made over a frequency range of 150kHz to 450mHz for the antennas shown in Figure One. In actual practice several different antennas would be used to cover such a wide range of frequencies but the effects under discussion are best illustrated when the same antennas are used for all frequencies under consideration.

A reciprocity check was conducted on antenna pairs by applying 1000 Volts to the terminals of one antenna and computing the current into 50 Ohms at the terminals of the undriven antenna and then reversing the process. Since the driving voltages and terminations are identical the currents at each antenna in the receiving mode should be identical. The ratios, in dB, of the currents received by the antennas are shown in Table #1. Good agreement is shown for the dipole (Antenna #2) and the one meter monopole (Antenna #1, DUT) in most parts of the spectrum under consideration. For the two antennas 10 meters apart (Antennas #1 & #3) the agreement is not as good at 250 and 450 mHz. This effect will be discussed later.

TABLE #1

RECIPROCITY BETWEEN ANTENNA PAIRS
(CURRENTS FOR 1000 VOLT DRIVES)

FREQUENCY IN mHz	* I1/I3, ANTENNA #1 & ANTENNA #3	* I2/I1, ANTENNA #1 & ANTENNA #2
0.15	0.0 dB	0.0 dB
4.0	0.0	0.0
10.0	0.0	0.0
50.0	-0.2	-0.6
100.0	-0.2	-0.87
250.0	-1.9	-0.89
450.0	-1.5	+0.15

* ANTENNAS TERMINATED IN 50 Ω

An attempt to predict the RF fields generated by a one meter length of twisted pair was made by the author in 1967. Referring to Figure One, Antenna #1 was the twisted pair terminated in 50 ohms, while Antenna #2 was the antenna used to monitor the fields from Antenna #3 that were produced for the RF immunity testing of Antenna #1. Antenna #2 was also used to measure the RF fields produced by currents flowing in the twisted pair. A voltage at the terminals of Antenna #2 was computed that would generate the same field at Antenna #1 as that produced by Antenna #3 during immunity testing. The current in Antenna #1 produced by the field from Antenna #3 was measured, and a calculation was made of the voltage at Antenna #2 that would produce the same current at Antenna #1, but corrected for near field effects.

Reciprocity was then invoked to calculate the current at Antenna #2 caused by a known voltage at the terminals of Antenna #1. Single frequency measurements were made for this experiment and the results, in terms of the units for which the instruments were calibrated, are shown in the graph of Figure Three.

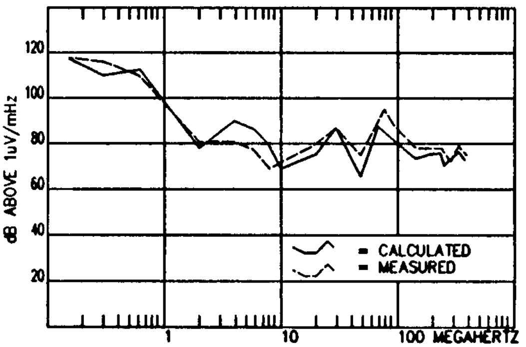

FIG. 3
ACTUAL AND PREDICTED RADIATED GENERATION IN 1967

I was unable to duplicate these results using MININEC. When the orthogonal near field component was used as a generated field reasonable agreement was achieved, but since this field is at right angles to the measurement dipole the happy results from twenty three years ago are probably a fluke. Difficulties in the interpretation of the near and far fields of the three antennas of Figure One further complicate that experiment.

These difficulties are overcome if the same antenna is used for measurement of the RF fields emitted by the DUT that is used to generate the fields used to determine the RF immunity of the DUT. I have not conducted such an experiment but the results of Table #1 suggest that good results could be achieved for certain frequency ranges.

Problems With Antenna Factors Using Antennas in the Near Field

A series of MININEC computations were run on the antennas shown in Figure One. In all cases 1000 volts was applied to the terminals of the driven antenna while the terminals of the receiving antenna, or antennas, were terminated in 50 Ohms. The interaction of Antennas #1 & #2 in the presence of fields generated by Antenna #3 was computed. The currents at the terminals of Antenna #1 were computed with and without the presence of Antenna #2.

The effect of the measurement dipole upon the DUT is much greater than the effect of the DUT upon the measurement dipole. The effect upon the dipole is less than .5 dB for frequencies below 50 mHz. The measurement dipole causes a variation of +3 to -1 dB in the currents in the DUT.

An antenna factor was computed for the dipole (Antenna #2) by generating a field at a distance of 3000 metres. Two measures show the degree to which far field conditions are achieved. The first is the requirement that the ratio of the electric and magnetic field components of the electromagnetic wave front of a plane wave is near the impedance of free space (377 Ohms). The second is that the computed near field component parallel to the receiving antenna is close to the computed far field. Table Two shows that far field conditions were obtained for the computation of the antenna factor for Antenna #2. An anomaly in the impedance at 250 mHz will be discussed later.

TABLE #2

DIPOLE (ANT.#2) CALIBRATION OVER GROUND PLANE AT 3000 METRES FROM 2 METRE MONOPOLE SOURCE

FREQUENCY IN mHz	FAR FIELD AT 3000 METRES	K*	$Z_0 = E_2/H$	FAR FIELD / VERT.NEAR FIELD
0.15	21.36 dBu	136.48 dB	373 OHMS	1.005
4.0	78.50	87.91	378	0.999
10.0	95.06	79.90	380	0.999
50.0	123.50	64.77	387	0.998
100.0	126.00	54.37	375	0.999
250.0	128.70	57.34	467	0.9916
450.0	121.20	67.68	367	1.030

* FAR FIELD DIVIDED BY $I_{50\Omega}$

The accuracy of the dipole in measuring the fields from the DUT at a distance of 0.3 meters (Figure One) is shown in Table Three. Here the comparison is made between the MININEC computed field from the DUT at 0.3 meters and the field computed by multiplying the current at the terminals (50 Ohm termination) of the dipole by the antenna factors shown in Table Three. The near field given by MININEC at 0.3 meters is the exact field at that location computed without using the simplifying assumptions used to compute the far field. The use of the antenna factor gives a result that is within about 1.6 dB of the actual field over a frequency range of 4 to 250 Mhz. It can also be seen from Table Three that the use of the far field antenna factor does not give a realistic assessment of the expected far field of the DUT at such a close distance.

TABLE #3

FAR FIELD ANT.#2 ERROR 0.3 METRES FROM DUT (ANT.#1) OVER A GROUND PLANE

FREQUENCY IN mHz	ERROR IN MEASURING NEAR FIELD	ERROR IN MEASURING FAR FIELD
0.15	+35.30 dB	+113.0 dB
4.0	-1.55	+36.0
10.0	-1.53	+20.0
50.0	-1.04	-4.1
100.0	-0.30	-12.2
250.0	+1.55	+4.85
450.0	+8.35	-3.80

The results from making the measurement at three metres from the DUT are shown in Table Four. It can be seen that the radial, or orthogonal, component of the field predominates (+4 dB or greater) over the vertical component of the electric field for frequencies of 10 mHz or less. The ability to infer the far field from the measurements of the DUT at this distance is somewhat impaired (+17 to -2 dB from 4 to 250 mHz) although the accuracy is within about 2 dB from 20 to 200 mHz. The ability to measure the vertical component of the near field, or actual field, is better than 0.5 dB from 4 to 100 mHz.

If the dipole is moved to a distance of 10 metres from a source antenna two metres long (monopole over a ground plane), the accuracy of the dipole far field measurements is within 1.4 dB from 10 to 100 mHz (Table Five). The accuracy of the near field dipole measurement is nearly 0 dB over the same range.

TABLE #4

ELECTRIC FIELD MEASUREMENT ERROR
ANT.#2 3 METRES FROM ANT.#1

FREQUENCY IN mHz	ERROR IN READING FAR FIELDS @ 3 M	ERROR IN READING NEAR FIELDS @ 3M	RADIAL NF / VERTICAL NF
0.15	+95.40 dB	+20.50 dB	+5.82 dB
4.0	+17.70	+0.38	+4.28
10.0	+0.10	+0.44	+6.67
50.0	-2.20	+0.07	-7.20
100.0	-2.30	+0.05	-8.60
250.0	+6.00	-1.63	-4.57
450.0	-6.60	-2.30	-8.44

TABLE #5

ANT.#2 MEASUREMENT ERROR 10 METRES FROM ANT.#3

FREQUENCY IN mHz	MININEC VERTICAL NEAR FIELD	NEAR FIELD MEASUREMENT ERROR	MININEC FAR FIELD	FAR FIELD MEASUREMENT ERROR
0.15	130.3 dBu	+40.00 dB	50.9 dBu	+79.40 dB
4.0	109.3	+20.00	108.1	+21.20
10.0	123.5	0.0	144.6	-1.10
50.0	152.6	0.0	153.0	-0.39
100.0	154.0	0.0	155.6	-1.40
250.0	150.6	+1.35	158.2	-7.60
450.0	152.2	+8.00	150.8	+1.40

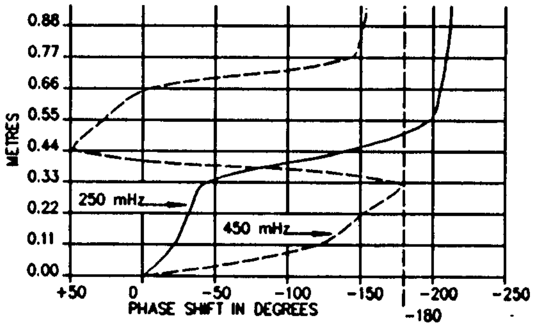

FIG. 4
ANT.#1 CURRENT PHASE SHIFT

Discussion of Anomalies

At 250 and 450 mHz there is considerable phase shift in the antenna currents as a function of antenna length for one and two metre monopoles. Figure Four shows the phase shift of the currents as a function of height above the ground plane, at 250 and 450 mHz, for the one metre Antenna #1 used to simulate a DUT. The current over a large part of the antenna is out of phase with the current in the other parts of the antenna.

The phase shift of the current in the two meter high Antenna #3 is shown for the same two frequencies in Figure Five. Here the effect is even more pronounced.

The contributions to the fields by the current moments of the portions of the antenna that are mutually out of phase can be represented by elements of a colinear array such as is shown in Figure Six. In the array far field of this colinear array the path lengths to its various elements are equal and parallel. The field is the sum of the current moments of all the elements and varies inversely as the distance from the antenna. Closer to the antenna the path lengths from the various elements are not equal.

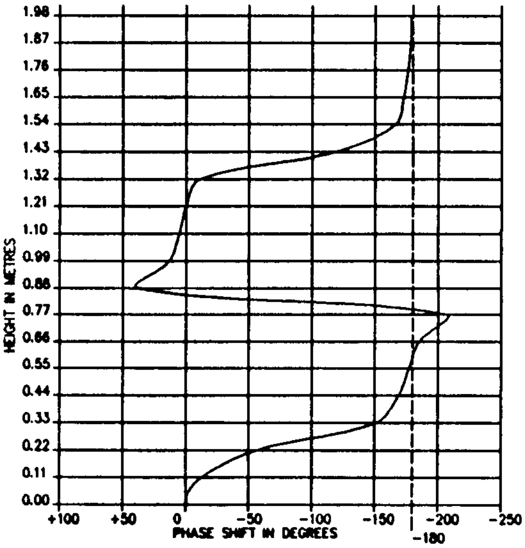

FIG. 5A
ANT.#3 CURRENT PHASE SHIFT AT 250 mHz

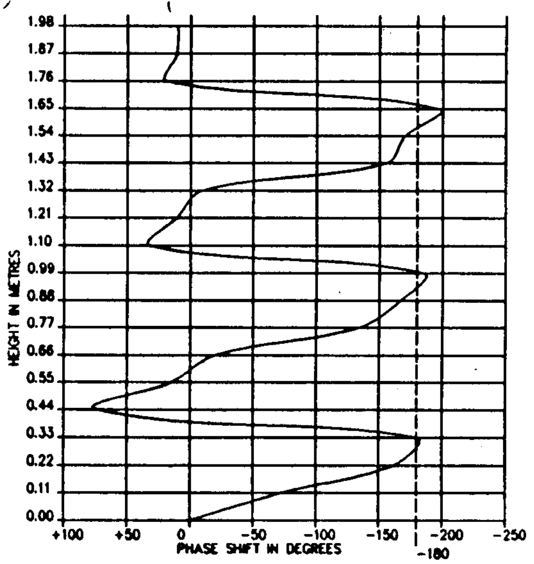

FIG. 5B
ANT.#3 CURRENT PHASE SHIFT AT 450 mHz

For Antenna #3 this means that the phase shifts of the contributions to the total field from different parts of the antenna will be a function of the distance from the antenna. This, and the effect of reflections from the ground plane, cause the fields from the antenna to undergo a variation with distance, near the antenna, that is quite different from that experienced by a far field plane wave. This effect is shown in Figure Seven for Antenna #3.

This variation between the actual fields and the far fields, as a function of distance, results in errors in the measurements of the far fields of the DUT at these frequencies, in certain circumstances, when the lengths of interconnecting cables are between one and two metres.

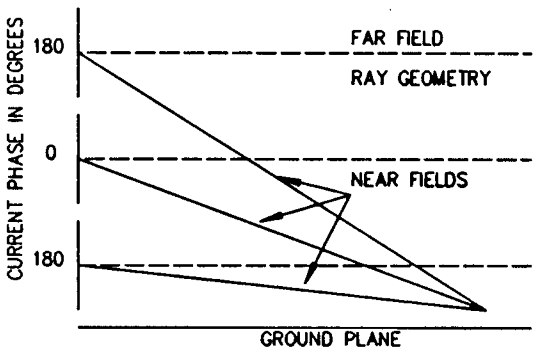

FIG. 6
EQUIVALENT COPLANAR ARRAY
FOR ANT.#3 AT 250 mHz

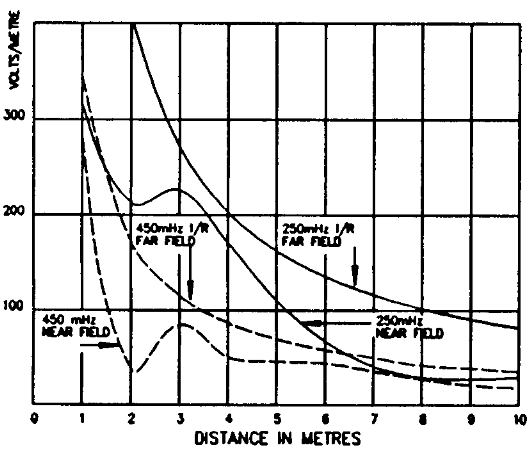

FIG. 7
FIELDS FROM ANT.#3

This can also affect the relationship between the electric and magnetic field components and cause the apparent free space impedance anomalies mentioned above. At 250 mHz the far field measurement error at distances between seven and ten metres from a source two metres in length is between -9 and -11 dB. At 450 mHz the error two metres from the same source is -13 dB and about -5.5 dB at distances of 9 to 10 metres.

CONCLUSION

The reciprocity principle can be used to compute the current or voltage, at the terminals of the antenna used to generate the RF immunity test fields, that would be created by the RF emissions from the device under test. The following parameters must be known to make the reciprocity computation: (1) the currents induced in the device during RF immunity testing, as a function of frequency; (2) the effective load impedance, as a function of frequency, at the point of RF current measurement in the device; (3) the voltage generated by the device {at the same point as (2)} as a function of frequency, that would produce the RF emissions from the device; (4) the terminal impedance of the measurement antenna as a function of frequency. A further requirement is that the computations only apply to the antenna, in the same orientation, at the same distance, that is used to generate the immunity test fields. The procedure becomes less accurate at frequencies above about 200 mHz when the length of the cables, or other wires used in the device to carry current, are over a meter in length. In many cases assembling the data for the reciprocity computation would be more trouble than performing the radiated emissions measurements.

The meaning of the current at the terminals of a measurement antenna in relation to the far field RF emissions of a device is very uncertain when the length of the device wiring is long enough to be a large proportion of a wavelength. Errors, at frequencies around 250 mHz, of 10 dB can be expected at distances of 10 metres or more from the device under test. This error is independent of the nature of the measurement antenna. In many circumstances the measurement antenna is never in the far field.

These conclusions are based on the assumption that the results of the analysis of the simplified geometry discussed in this paper can be extrapolated to other, more complex, geometries. What is implied herein is a suggestion that computations and measurements be performed on antennas and devices in an actual test environment. The essential point is that numerical codes such as MININEC can be used on a mini-computer, or PC, to gain valuable insight into the performance of antennas, and the nature of the RF fields, involved in EMC measurements.

References

[1] King, R.W.P.:The Theory of Linear Antennas, Harvard University Press, Cambridge, Massachusetts, USA, (1956), Page 568, Equation (1).

[2] Ibid. page 465, Fig. 4.3b

[3] Ibid. page 466, Equation (3)

[4] Rockway, J.W.; Logan, J.C.; Tam, D.W.S.; Li, S.T.: The MININEC System: Microcomputer Analysis of Wire Antennas, Artech House, Boston Ma., USA, 1988.